Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[5 parts, 2 points each]** Let $\Sigma = \{a, b, c\}$; we define the following languages:

$$F = \{w : \text{ the number of } a\text{'s equals the number of } b\text{'s}\}$$
$$G = \{w : \text{ the number of } b\text{'s equals the number of } c\text{'s}\}$$
$$H = \{w : \text{ all } a\text{'s in } w \text{ appear before all } c\text{'s}\}$$

(a) Give an example of a word $w \in F - G$.

Examples: c, ba, $abab$, $abcc$

Anything where $\#a = \#b$ but $\#c \neq \#b$.

(b) Give two examples of words in $F \cap G$.

Examples: λ, abc, $abaccb$

Anything where $\#a = \#b = \#c$.

(c) True or False: $FF = F$. If True, give an argument justifying your claim. If False, give an example of a word w that belongs to exactly one of the languages in $\{FF, F\}$ and is omitted from the other.

$\boxed{\text{True:}}$ If ~~xy~~ $w \in FF$ where $w = xy$ with $x \in F$ and $y \in F$, then both ~~pan~~ x and y have the same # of a's and b's, and so does w.

If $w \in F$, then $w = w\lambda \in FF$ since both w and λ have the same # of a's and b's.

F: equal #a's = #b's G: #b's = #c's H: all a's before all c's

(d) True or False: $FH = H$. If True, give an argument justifying your claim. If False, give an example of a word w that belongs to exactly one of the languages in $\{FH, H\}$ and is omitted from the other.

$\boxed{\text{False:}}$ let $w = ca$. Note that $w = xy$ where $x = c$ and $y = a$.

Since x has the same # of a's as b's, we have $x \in F$.

Since all a's appear before all c's in y, we have $y \in H$.

Therefore $w \in FH$. But $w \notin H$ since the c comes before the a in w.

(e) True or False: $F \cup G \subseteq FG$. If True, give an argument justifying your claim. If False, give an example of a word w that belongs to $F \cup G$ but does not belong to FG.

$\boxed{\text{True}}$. Suppose $w \in F \cup G$. If $w \in F$, then we have $w = w\lambda \in FG$ since $w \in F$ and $\lambda \in G$. If $w \in G$, then we have

$$w = \lambda w \in FG$$

since $\lambda \in F$ and $w \in G$.