

Directions: You may work to solve these problems in groups, but all written work must be your own. **Show your work;** See “Guidelines and advice” on the course webpage for more information.

1. Let $A = \{1, 2\}$, let $B = \{2, 3, (4, 5)\}$, and let $C = \{\emptyset, \{1, 2\}, \{2, 1\}\}$, $D = \{2, 3, (5, 4)\}$.
 - (a) Determine $|A|$, $|B|$, and $|C|$.
 - (b) Determine the sets $A \times B$ and $A \times C$.
 - (c) True or False: $C \subseteq \mathcal{P}(A)$
 - (d) True or False: $A \subseteq \mathcal{P}(C)$
 - (e) Determine the set $B \triangle D$.

2. A *bitstring* is an ordered list of zeros and ones; for example, 0110 and 10100 are bitstrings of lengths 4 and 5, respectively. As a special case, we use ε to denote the empty bitstring, which has length 0.
 - (a) Show that the set of all bitstrings of finite length is countable.
 - (b) Is the set of all bitstrings of infinite length countable? Justify your answer.

3. Let L be the set of lines in the plane which (1) intersect the x -axis and the y -axis at integral points, and (2) do not contain the origin $(0, 0)$. For example, L contains the graph of $y = \frac{2}{3}x + 4$ since this line meets the y -axis at $y = 4$ and the x -axis at $x = -6$. But L does not contain the graph of $y = 4x + 3$, since this line meets the x -axis at $x = -\frac{3}{4}$, and $-\frac{3}{4}$ is not an integer, and L does not contain the graph of $y = x$, since this line passes through the origin $(0, 0)$. Is the set L countable? Justify your answer.