Directions: Show your work. You may work to solve these problems in groups, but all written work must be your own. See "Guidelines and advice" on the course webpage for more information.

1. How many ways are there to partition the integers $\{1, 2, 3, \ldots, 2n\}$ into pairs? For example, when $n = 2$, there are 3 ways: $\{\{1, 2\}, \{3, 4\}\}, \{\{1, 3\}, \{2, 4\}\}, \{\{1, 4\}, \{2, 3\}\}$. Note that the order of the pairs is not important (so $\{\{3, 4\}, \{1, 2\}\}$ is the same is $\{\{1, 2\}, \{3, 4\}\}$ and the order of within pairs is unimportant (so $\{\{2, 1\}, \{3, 4\}\}$ is the same as $\{\{1, 2\}, \{3, 4\}\}$).

   Hint: There are several ways to solve this. Here is one nice way. Count the number of permutations of $\{1, 2 \ldots, 2n\}$ of length $2n$ in two different ways: one directly, and the other using the rule of product (where one stage involves the unknown quantity).

2. Let $f(n)$ be the number of ordered lists of positive integers that sum to n. For example:

n	$f(n)$	Lists with sum n
1	1	(1)
2	2	(1, 1), (2)
3	4	(1, 1, 1), (2, 1), (1, 2), (3)

   It is pretty easy to guess a formula for $f(n)$. It is more difficult to give an argument that shows your formula for $f(n)$ is correct. Argue that your formula is correct by using a graphical representation of the integer n as a row of n dots.

3. Poker hands.

   (a) A *full house* is a poker hand in which 3 cards have the same rank and the other 2 also have the same rank (e.g. 3 of hearts, 3 of spades, 3 of clubs, 7 of diamonds, 7 of clubs). How many poker hands are full houses? What are the odds of being dealt a full house from a freshly shuffled deck?

   Hint: Resist the temptation to look up the answer online. It is not allowed *and* it defeats the purpose of the question.

   (b) A *near flush* is a poker hand in which 4 cards belong to a single suit and the remaining card belongs to a different suit. How many poker hands are near flushes? What are the odds of being dealt a near flush from a freshly shuffled deck?

4. How many ways are there to arrange the letters of the word 'SUSPICIOUS':

   (a) with no additional restrictions?

   (b) if no two S's are consecutive?

5. How many rectangles are created when n horizontal lines intersect n vertical lines? (Note: a square is a rectangle, so squares are included.) For example, when $n = 3$, there are 9 rectangles, one of which is highlighted below: