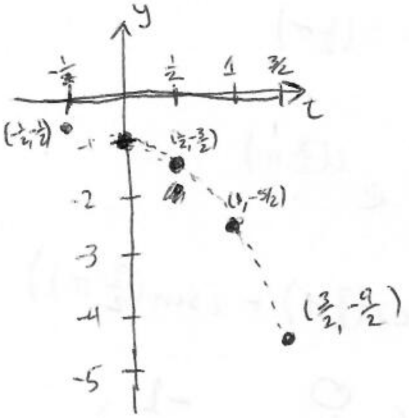
Name: Solutions.

Directions: Show all work. No credit for answers without work.

1. Consider the IVP $y' = 2y + 1$ with $y(0) = -1$.

   (a) **[3 points]** Use Euler's Method with step size $h = 1/2$ to approximate the solution at $t = 1/2$, $t = 1$, and $t = 3/2$.



$x_0 = 0$

$y_0 = -1$

$x_1 = x_0 + h = \frac{1}{2}$

$y_1 = y_0 + mh = -1 + (2(-1)+1)\frac{1}{2} = -1 + (-2+1)\frac{1}{2} = -1 - \frac{1}{2} = -\frac{3}{2}$

$x_2 = x_1 + h = \frac{1}{2} + \frac{1}{2} = 1$

$y_2 = y_1 + mh = -\frac{3}{2} + (2(-\frac{3}{2})+1)\frac{1}{2} = -\frac{3}{2} + (-3+1)\frac{1}{2} = -\frac{3}{2} - 1 = -\frac{5}{2}$

$x_3 = x_2 + h = 1 + \frac{1}{2} = \frac{3}{2}$

$y_3 = y_2 + hm = -\frac{5}{2} + \frac{1}{2}(2(-\frac{5}{2})+1) = -\frac{5}{2} - \frac{5}{2} + \frac{1}{2} = -5 + \frac{1}{2} = \frac{-9}{2}$

t	$\frac{1}{2}$	1	$\frac{3}{2}$
$y(t)$ approx	$-\frac{3}{2}$	$-\frac{5}{2}$	$-\frac{9}{2}$

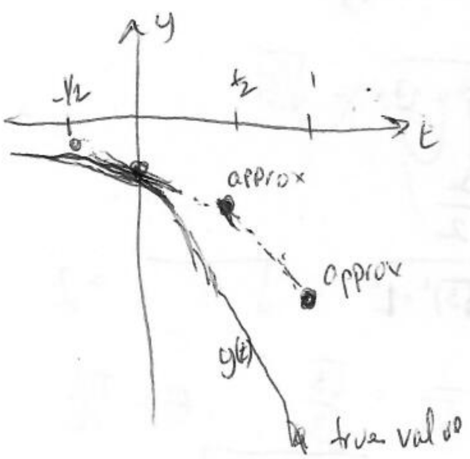
   (b) **[2 points]** Extend Euler's Method in a natural way to approximate the solution at $t = -1/2$.

$x_{-1} = x_0 - h = 0 - \frac{1}{2} = -\frac{1}{2}$

$y_{-1} = y_0 - hm = -1 - \frac{1}{2}(2(-1)+1) = -1 + 1 - \frac{1}{2} = -\frac{1}{2}$.

So $y(-\frac{1}{2})$ is approximated by $-\frac{1}{2}$.

   (c) **[1 point]** Are the approximations found in parts (a) and (b) larger than, smaller than, or equal to the corresponding true values $y(-1/2)$, $y(1/2)$, $y(1)$, $y(3/2)$? (Your answer may vary from approximation to approximation.)



In all cases, the approximations are $\boxed{\text{larger}}$ than the true values. The approximation assumes a constant value for y' over short windows of size h, but the true value of y' is decreasing, meaning that y goes down faster than the approximation.

2. **[2 parts, 1 point each]** Convert the following complex numbers into Cartesian form $a + bi$.

(a) $\frac{3+i}{-2+5i}$

$$\frac{(3+i)(-2-5i)}{(-2+5i)(-2-5i)} = \frac{-6-17i-5i^2}{4-25i^2}$$

$$= \frac{-1-17i}{29}$$

$$= \boxed{-\frac{1}{29} - \frac{17}{29}i}$$

(b) $e^{(\pi+i)(\pi/2+i)}$

$$= e^{\pi^2/2 + i(\pi/2+\pi) + i^2}$$

$$= e^{\pi^2/2 - 1 + i\left(\frac{3}{2}\pi\right)}$$

$$= e^{\pi^2/2 - 1} \cdot e^{i\left(\frac{3}{2}\pi\right)}$$

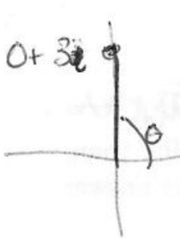
$$= e^{\pi^2/2 - 1}\left(\cos\left(\tfrac{3}{2}\pi\right) + i\sin\left(\tfrac{3}{2}\pi\right)\right)$$

$$= e^{\pi^2/2 - 1}\left(0 + (-1)i\right)$$

$$= \boxed{0 - e^{(\pi^2/2)-1}\, i} = \boxed{-e^{\pi^2/2-1}\, i}$$

3. **[2 parts, 1 point each]** Convert the following complex numbers into polar form $re^{i\theta}$.

(a) $3i$

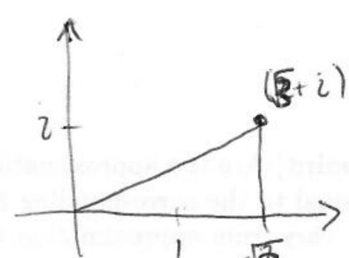


$$r = \sqrt{0^2+3^2} = 3$$

$$\theta = \pi/2$$

$$3i = \boxed{3e^{(\pi/2)i}}$$

(b) $\cancel{2}\sqrt{3} + i$



$$\cancel{r = \sqrt{2^2+1^2} = \sqrt{5}}$$

$$\cancel{\tan\theta = \frac{1}{2}}$$

$$r = \sqrt{(\sqrt{3})^2+1^2} = \sqrt{4} = 2$$

$$\tan\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}, \quad \theta = \pi/6$$

$$\sqrt{3}+i = \boxed{2e^{(\pi/6)i}}$$