Name: ______________________________

Directions: Show all work. Answers without work generally do not earn points.

1. [**10 points**] Let $\Sigma = \{a, b\}$. Construct a simple NFA for the language $\{w \mid w \text{ ends with } aaa \text{ or } bbb\}$.

2. [**3 parts, 2 points each**] Let $\Sigma = \{a, b\}$. Let

$$A = \{w \mid w \text{ has at least as many } a\text{'s as } b\text{'s}\}$$
$$B = \{w \mid w \text{ has at least as many } b\text{'s as } a\text{'s}\}.$$

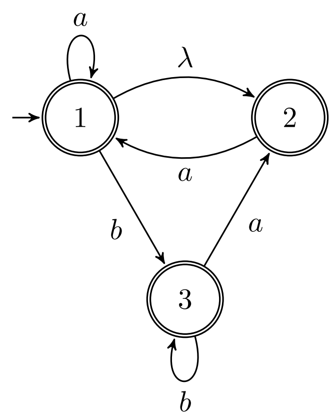
For each language below, give a DFA if the language is regular or write "not regular" if the language is not regular.

(a) A

(b) B

(c) AB

3. Let $\Sigma = \{a, b\}$ and let N be the following NFA.



(a) **[10 points]** Convert N to an equivalent DFA.

(b) **[2 points]** What is the shortest word that is *not* accepted by N?

(c) **[2 points]** Give a simple, English description of $L(N)$.

4. **[5 points]** Let N be an NFA that recognizes a language A. Describe how to use N to make a new NFA or DFA which recognizes the complement language $\{w \mid w \notin A\}$.

5. **[2 parts, 5 points each]** Let $\Sigma = \{a, b\}$, and let $A = \{w \mid w$ has an odd number of a's$\}$.

   (a) Give an NFA for the language AA.

   (b) Convert the NFA to a DFA and simplify.

6. **[2 parts, 8 points each]** Let $\Sigma = \{a, b\}$, and let $A = \{w \mid \text{the length of } w \text{ is not a multiple of } 3\}$, and let $B = \{w \mid w \text{ has an odd number of } b\text{'s}\}$.

   (a) Give an NFA for the language AB.

   (b) Convert the NFA to a DFA and simplify.

7. **[5 points]** Draw a 5-vertex graph (without loops and without parallel edges) in which one vertex has degree 4, one vertex has degree 3, two vertices have degree 2, and one vertex has degree 1.

8. **[5 points]** Give a simple argument that there is no 15-vertex graph (without loops and without parallel edges) in which two vertices have degree 14 and one vertex has degree 1.

9. **[5 points]** Find an 8-vertex graph which contains C_5, C_6, and C_8 as subgraphs but does not contain cycles of any other lengths.

10. For $n \geq 2$, let G_n be the graph whose vertices are the subsets of $\{1, 2, \ldots, n\}$ of size 2 and where two vertices are adjacent if and only if they have non-empty intersection. For example, in G_5, the vertices $\{3, 4\}$ and $\{3, 5\}$ are adjacent but $\{3, 4\}$ and $\{1, 5\}$ are not adjacent.

    (a) [**6 points**] Draw G_2, G_3, and G_4.

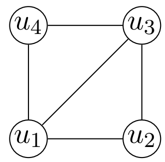
    (b) [**2 points**] Give a formula for the number of vertices in G_n.

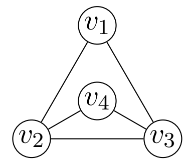
    (c) [**2 points**] Give a formula for the degree of a vertex in the regular graph G_n.

    (d) [**2 points**] Use parts (b) and (c) to give a formula for the number of edges in G_n.

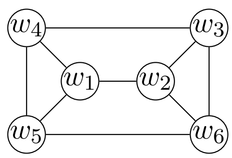
11. **[3 parts, 4 points each]** For each pair of graphs below, decide if the graphs are isomorphic. If they are isomorphic, give a table. If not, argue why not.

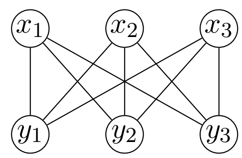
(a)





(b)





(c)