Name: _____

Directions: Show all work. No credit for answers without work.

1. **[2 points]** Let $\Sigma = \{0, 1\}$. Determine $|\Sigma^5|$.

2. Let $\Sigma = \{0, 1\}$ and let A be the language defined recursively as follows:

   (1) $\lambda \in A$.

   (2) If $x \in A$, then $0x \in A$.

   (3) If $x \in A$, then $x1 \in A$.

   (a) **[2 points]** List all strings in A of length at most 2.

   (b) **[1 point]** Give a simple, non-recursive description of A.

   (c) **[1 point]** Give a formula for the number of strings in A of length n.

3. **[4 parts, 1 point each]** Let $\Sigma = \{0, 1, 2\}$. Let A be the language of all strings over Σ with an equal number of 0's and 1's, and let B be the language of all strings over Σ with an equal number of 1's and 2's. For example, if $w = 1202112$, then the number of occurrences of 0, 1, and 2 in w is 1, 3, and 3 respectively. Consequently $w \notin A$ and $w \in B$.

   (a) Give a simple, English description of the language $A \cap B$.

   (b) Determine the number of strings in $A \cap B$ of length 10.

   (c) True or false: if x and y are both in A, then $xy \in A$.

   (d) Give an example of a string w over Σ that has length 2 such that $w \in AB$ and $w \notin BA$.