Name: _____

Directions: Show all work. No credit for answers without work.

1. **[4 parts, 1 point each]** True or False? Write the whole word. (No work necessary.)

   (a) For all sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

   (b) For all sets A and B, it is the case that $A \times B = B \times A$.

   (c) For all sets A and B, it is the case that $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.

   (d) For all sets A and B, it is the case that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

2. Let A be the set of all finite subsets of $\mathbb{N}$. (Recall $\mathbb{N} = \{0, 1, 2, 3 \ldots\}$. For example, $\{4, 8, 10\} \in A$ and $\{3, 6, 9, 12, 15, 18\} \in A$.)

   (a) **[2 points]** Show that A is countable by describing, in English sentences, a way to list the elements of A.

   (b) **[1 point]** In addition to the English description in part (a), explicitly give the first 10 elements of A in your list.

3. **[3 parts, 1 point each]** Let A be the set of all subsets of $\{1, 2, 3, \ldots, n, n+1\}$ of size 3.

   (a) Determine $|A|$.

   (b) For $k \leq n$, let B_k be the number of subsets of $\{1, 2, 3, \ldots, n, n+1\}$ of size 3 whose maximum element equals $k+1$. (For example, $\{2, 3, 7\} \in B_6$ and $\{1, 4, 7\} \in B_6$ since both sets have 7 as their maximum.) Determine $|B_k|$.

   (c) Give a simple formula for $\sum_{k=0}^{n} \binom{k}{2}$.