Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[4 parts, 1 point each]** True or False? Write the whole word. (No work necessary.)
   (a) For all sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$. True
   (b) For all sets A and B, it is the case that $A \times B = B \times A$. False
   (c) For all sets A and B, it is the case that $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$. False
   (d) For all sets A and B, it is the case that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$. True

2. Let A be the set of all finite subsets of $\mathbb{N}$. (Recall $\mathbb{N} = \{0, 1, 2, 3 \ldots\}$. For example, $\{4, 8, 10\} \in A$ and $\{3, 6, 9, 12, 15, 18\} \in A$.)
   (a) **[2 points]** Show that A is countable by describing, in English sentences, a way to list the elements of A.

<u>Alt. soln</u>:

We can also list the ~~els~~ sets of A according to the sum of their elements.

List the sets according to their <u>maximum</u> <u>elements</u>.

(Since $\emptyset$ does not have a maximum element, we list it first.)

There are 2^k subsets of $\mathbb{N}$ with maximum element k, and since this is finite every set in A eventually appears on the list.

   (b) **[1 point]** In addition to the English description in part (a), explicitly give the first 10 elements of A in your list.

$$A = \{\emptyset, \underbrace{\{0\}}_{\text{max elt } 0}, \overbrace{\{1\}, \{0,1\}}^{\text{max elt } 1}, \underbrace{\{2\}, \{0,2\}, \{1,2\}, \{0,1,2\}}_{\text{max elt } 2}, \overbrace{\{3\}, \{0,3\}}^{\text{max elt. } 3}, \ldots\}$$

3. **[3 parts, 1 point each]** Let A be the set of all subsets of $\{1, 2, 3, \ldots, n, n+1\}$ of size 3.

   (a) Determine $|A|$.

$$|A| = \boxed{\binom{n+1}{3}}$$

   (b) For $k \leq n$, let B_k be the number of subsets of $\{1, 2, 3, \ldots, n, n+1\}$ of size 3 whose maximum element equals $k+1$. (For example, $\{2, 3, 7\} \in B_6$ and $\{1, 4, 7\} \in B_6$ since both sets have 7 as their maximum.) Determine $|B_k|$.

To form elts in B_k, select a pair from $\{1, 2, \ldots, k\}$

~~$|B_k| =$~~

and ~~add k+1~~ ~~elt~~ add $k+1$ as a member to complete the triplet.

So $|B_k| = \boxed{\binom{k}{2}.}$

   (c) Give a simple formula for $\sum_{k=0}^{n} \binom{k}{2}$.

By rule of sum, $|A| = \sum_{k=0}^{n} |B_k|$. So:

$$\sum_{k=0}^{n} \binom{k}{2} = \sum_{k=0}^{n} |B_k| = |A| = \boxed{\binom{n+1}{3}}$$