**Math375** **Quiz 2** **August 27, 2015**

Name: Solutions

**Directions:** Show all work. No credit for answers without work.

1. How many ways are there to arrange the letters of 'SLEEPLESS':

(a) [**2 points**] with no additional restrictions?

#S: 3

#L: 2

#E: 3

#P: 1

total # letters: 9

$$\# \text{ of ways} = \frac{9!}{(3!)(3!)(2!)} = \boxed{5,040}$$

(b) [**1 point**] beginning with an L?

Count arrangements of "SEEPLESS".

$$\# \text{ ways} = \frac{8!}{(3!)(3!)} = \boxed{1120}$$

(c) [**1 point**] beginning with an L and ending with some letter besides L?

#(ways beginning with L) = (#ways beginning and ending with L) + X

$$1120 = \frac{7!}{(3!)(3!)} + X$$

$$1120 = 140 + X, \quad x = \boxed{980}$$

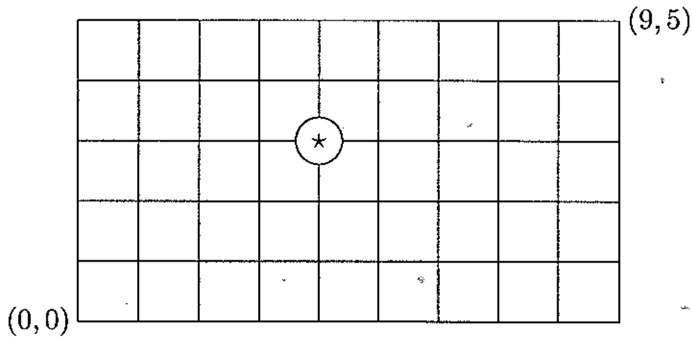
(d) [**1 point**] if all three E's are to the left of all three S's?

Combine E's and S's into a new symbol ?

Count arrangements of ?L??PL???; then ~~fill~~ replace ?'s with EEESSS in that order.

$$\# \text{ ways} = \frac{9!}{(6!)(2!)} = \frac{9 \cdot 8 \cdot 7}{2} = \boxed{252}$$

2. Lattice paths from $(0,0)$ to $(9,5)$. Recall that each step of a lattice path increases one of the coordinates by 1; geometrically, we either move one unit in the horizontal direction or 1 unit in the vertical direction.




(a) [**2 points**] How many lattice paths are there from $(0,0)$ to $(9,5)$?

Arrange 9 U's and 5 R's:

$$\frac{(9+5)!}{(9!)(5!)} = \frac{14\cdot 13\cdot 12\cdot 11\cdot 10}{5\cdot 4\cdot 3\cdot 2\cdot 1} = \boxed{2{,}002}$$

(b) [**2 points**] Suppose there is a prize (denoted by $\star$) at $(4,3)$. How many lattice paths visit $(4,3)$ and win the prize?

Stage 1: $(0,0)$ to $(4,3)$: $\frac{(4+3)!}{(4!)(3!)} = \frac{7\cdot 6\cdot 5}{3\cdot 2\cdot 1} = 35$ ways

Stage 2: $(4,3)$ to $(9,5)$: 5R, 2U; $\frac{7!}{(5!)(2!)} = 21$ ways.

Total # $= 35\cdot 21 = \boxed{735}$

(c) [**1 point**] How many lattice paths miss the prize at $(4,3)$?

(#paths) = (#paths that hit $(4,3)$) + $x$

$2002 = 735 + x$

$x = \boxed{1{,}267}$