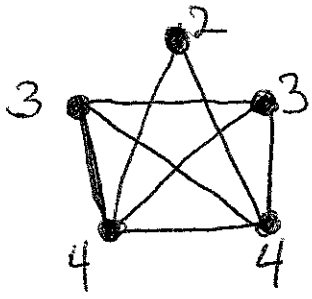
Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[2 points]** Draw a graph with 5 vertices in which 2 vertices have degree 4, and 2 vertices have degree 3, and 1 vertex has degree 2.



Note: ~~Parallel edges.~~ In a graph, parallel edges are not allowed.

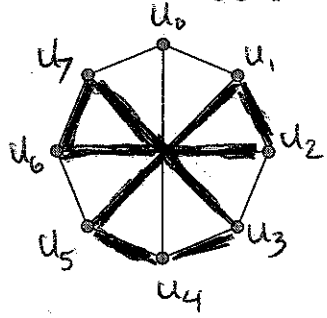
2. **[1 point]** Suppose G is graph with 20 vertices in which every vertex has degree 14. How many edges does G have?

$$|E(G)| = \frac{1}{2}\sum_{v\in V(G)} d(v) = \frac{1}{2}\sum_{v\in V(G)} 14 = \frac{1}{2}\cdot 20\cdot 14 = \boxed{140}$$

3. **[1 point]** Give a simple argument that there is no 7-regular graph with 15 vertices.

Such a graph would have $\frac{1}{2}\cdot 15\cdot 7$ edges. Since $\frac{15\cdot 7}{2}$ is not an integer, this is not possible.

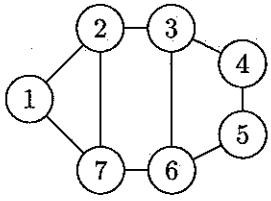
4. **[2 points]** Find a cycle of length 7 in the following graph:

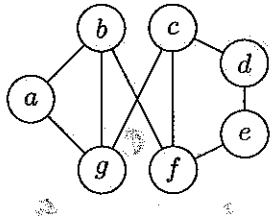


$u_1\ u_5\ u_4\ u_3\ u_7\ u_6\ u_2$

5. **[2 parts, 2 points each]** Decide whether the following pairs of graphs are isomorphic. If they are isomorphic, give the function that establishes the isomorphism. If not, explain why.

(a)





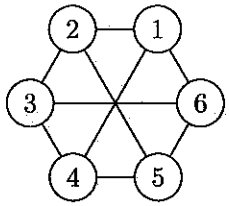
Isomorphic.

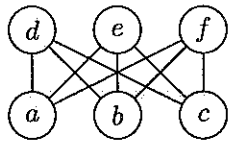
1	2	3	4	5	6	7
a	b	f	e	d	c	g

or

1	2	3	4	5	6	7
a	g	c	d	e	f	b

(b)





Isomorphic. Many ~~letters~~ functions work. What we need is $\{1,3,5\} \leftrightarrow \{a,b,c\}$, $\{2,4,6\} \leftrightarrow \{d,e,f\}$ or $\{1,3,5\} \leftrightarrow \{d,e,f\}$, $\{2,4,6\} \leftrightarrow \{a,b,c\}$.

For example:

1	2	3	4	5	6
b	d	a	e	c	f