Name: Solutions

Directions: Show all work. No credit for answers without work.

1. The house special of Joe's sandwich shop features a choice of 8 different toppings.

   (a) **[3 points]** Assuming no restrictions on the toppings, how many ways are there to order the house special?

Stage 1: Choose to include or omit topping 1 $(n_1 = 2)$

⋮

Stage 8: Choose to include or omit topping 8 $(n_8 = 2)$

So total # $= n_1 \cdot n_2 \cdot \cdots \cdot n_8 = \boxed{2^8 = 256}$

   (b) **[2 points]** Now, suppose that 4 of the toppings are *premium* toppings and the other 4 are *regular* toppings. There are no restrictions on the number of regular toppings that can be ordered, but at most 1 premium topping can be ordered. (Of course, it is possible to order the house special with no premium toppings.) Now how many ways are there to order the house special?

Stage 1: Choose at most one premium topping $(n_1 = 5)$

Four options to have 1 premium topping

One option to have 0 premium toppings

Stage 2: Choose regular toppings $n_2 = 2^4$

Two options in each of 4 separate mini-stages

So total # = $\boxed{5 \cdot 2^4 = 80}$

2. ATM Pin numbers

(a) **[3 points]** How many 4-digit ATM pin numbers do not contain a 5?

Stage 1: Choose the first a digit, which cannot be 5 $n_1 = 9$

⋮

Stage 4: Choose the fourth digit, which cannot be 5 $n_4 = 9$

So the total # is $\boxed{9^4 \text{ or } 6561}$.

(b) **[2 points]** How many 4-digit ATM pin numbers have a 5 in at least one position? For example, 5512 and 5555 are OK but 1234 is not.

(total # pins) = (#pins with a 5) + (# pins without a 5)

10,000 = (# pins with a 5) + 6561

(#pins with a 5) = 10,000 − 6561 = $\boxed{3439}$