Directions: You may work to solve these problems in groups, but all written work must be your own. **Show your work**; See "Guidelines and advice" on the course webpage for more information.

1. Show that when any edge is removed from K_5, the resulting subgraph is planar. Is this true for $K_{3,3}$?

2. Let G be a connected planar graph (without loops or parallel edges). One way of embedding G in the plane creates 53 regions, each of which has at least five edges on its boundary. Prove that G has at least 82 vertices.

3. Let G be a connected planar graph (without loops or parallel edges).

   (a) The complement of G, denoted $\overline{G}$, is the graph on $V(G)$ where u and v are adjacent in $\overline{G}$ if and only if they are non-adjacent in G. Prove that if $|V(G)| \geq 11$, then G or $\overline{G}$ is nonplanar. Hint: count edges in G and $\overline{G}$.

   (b) Find an 8-vertex graph G such that G and $\overline{G}$ are planar.