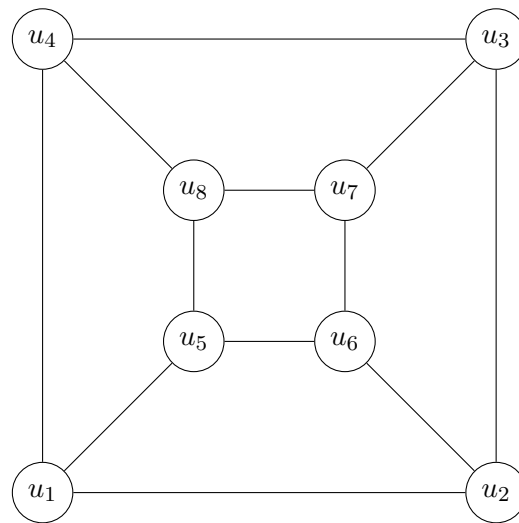


**Directions:** You may work to solve these problems in groups, but all written work must be your own. **Show your work;** See “Guidelines and advice” on the course webpage for more information.

1. Let  $\Sigma = \{0, 1\}$ . Let  $A$  be the language  $\{w \mid w \text{ is an integer in binary notation and } w \text{ is divisible by } 5\}$ . For example, 1010 represents  $1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 8 + 2 = 10$ , so  $1010 \in A$ . On the other hand, 01110 represents  $0 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 8 + 4 + 2 = 14$  so  $01110 \notin A$ . Give a DFA that recognizes  $A$ .
2. Show that if  $A$  and  $B$  are regular languages, then  $AB$  is a regular language. Hint: given a DFA  $M_1$  for  $A$  and a DFA  $M_2$  for  $B$ , describe how to use  $M_1$  and  $M_2$  to make an NFA for  $AB$ . Why is it enough to make an NFA?
3. Let  $G$  be the following graph.



- (a) Find two disjoint 4-cycles in  $G$ .
- (b) Find a 6-cycle in  $G$ .
- (c) Does  $G$  contain an 8-cycle? If so, describe one. If not, explain why not.
- (d) Show how to color the vertices red and blue so that no edge has two endpoints with the same color.
- (e) Does  $G$  contain a 5-cycle? If so, describe one. If not, explain why not.