

Directions: You may work to solve these problems in groups, but all written work must be your own. See “Guidelines and advice” on the course webpage for more information.

1. A sandwich shop offers 5 choices of bread, 3 choices of seasonings, 6 choices of meat. Also, the shop offers 8 toppings: lettuce, spinach, tomato, cheese, avocado, onion, cucumber, and bell pepper.
 - (a) How many different sandwiches can be ordered at the shop? If you ate one sandwich a day and never repeated orders, how long would it take you to try them all?
 - (b) The shop owner decides that having both lettuce and spinach at the same time makes the sandwich too green, and makes a rule that a sandwich cannot have both of these toppings. How many different sandwiches can be ordered now?
2. How many 5-digit ATM pin numbers have no repeated digits within distance 2 of one another? For example, 56759 counts because the pairs of 5’s are at distance 3 from one another. Similarly, 56756 counts, but 56765 does not count because the 6’s are too close to each other.
3. A library has n different books and two shelves. How many ways can they arrange the books on the shelves so that no shelf is empty?

For example, with 3 books (called 1, 2, and 3), there are 6 ways to do this with one book on the first shelf and two on the second: 1|23, 1|32, 2|13, 2|31, 3|12, 3|21. Since there are also 6 ways to do this with *two* books on the first shelf and *one* on the second, the answer when $n = 3$ is 12.