Math251 | **Quiz 8** | **October 28, 2013**

Name: Solutions

Directions: Show all work. No credit for answers without work.

1. [**2 points**] Find the first partial derivatives of the function $f(x,y,z) = x^2e^{x/y} - 2xyz^2$.

$$f_x = \boxed{2xe^{x/y} + \frac{x^2}{y}e^{x/y} - 2yz^2}$$

$$f_y = x^2e^{x/y}x(-y)^{-2} - 2xz^2 = \boxed{-\frac{x^3}{y^2}e^{x/y} - 2xz^2}$$

$$f_z = \boxed{-4xyz}$$

2. [**2 points**] Find the equation of the tangent plane to $z = y\ln(x^2+y^2)$ at the point $(0,1,0)$.

$$z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0). \qquad (x_0,y_0,z_0) = (0,1,0)$$

$$f_x = \frac{y}{x^2+y^2}\cdot 2x = \frac{2xy}{x^2+y^2}. \qquad f_x(0,1) = \frac{(2)(0)(1)}{0^2+1^2} = 0$$

$$f_y = \ln(x^2+y^2) + \frac{2y^2}{x^2+y^2}. \qquad f_y(0,1) = \ln(1) + \frac{2}{0^2+1^2} = 2$$

$$z - 0 = 0(x-0) + 2(y-1) \quad \text{or} \quad \boxed{z = 2y - 2}.$$

3. [**2 points**] The radius of a cylinder is measured to be 50 cm and its height is measured to be 100 cm. Both measurements have an error up to ± 0.5 cm. Use differentials to estimate the maximum possible error in the computed volume of the cylinder.

$$V = \pi r^2 h$$

$$dV = 2\pi rh\,dr + \pi r^2\,dh$$

$$\Delta V \approx dV = 2\pi(50)(100)\,dr + \pi(50)^2\,dh$$

$$\leq 2\pi(50)(100)\cdot\frac{1}{2} + \pi(50)^2\cdot\frac{1}{2}$$

$$= 2\pi(50)^2 + \pi(50)^2\cdot\frac{1}{2} = (50)^2\left[2\pi + \frac{\pi}{2}\right]$$

$$= \frac{5\pi}{2}\cdot 5\cdot 5\cdot 10\cdot 10 = \boxed{6{,}250\pi \text{ cm}^3}$$

4. **[2 points]** Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ where $z = x\sin(y^2)$, $x = s/t$, and $y = \cos t$. Note: you may leave your answer in terms of x, y, r, and s — no need to substitute to eliminate x and y.

- $$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s} = (\sin(y^2))\cdot\frac{1}{t} + (x\cos(y^2)\cdot 2y)\cdot 0$$

$$= \boxed{\frac{\sin(y^2)}{t}} = \boxed{\frac{\sin((\cos t)^2)}{t}}$$

- $$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t} = (\sin(y^2))\cdot\left(-\frac{s}{t^2}\right) + (x\cos(y^2)\cdot 2y)\cdot(-\sin t)$$

$$= \boxed{-\frac{s(\sin(y^2))}{t^2} - 2xy\cos(y^2)\sin(t)}$$

5. **[2 points]** If $z = f(x-y)$ and f is differentiable, show that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.

- Let $u = x - y$, so $z = f(u)$, $u = x - y$. One intermediate variable.

- $$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}$$

$$= \frac{\partial z}{\partial u}\cdot 1$$

- $$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} = \frac{\partial z}{\partial u}\cdot(-1)$$

- $$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} - \frac{\partial z}{\partial u} = 0$$