Math251 **Quiz 5** **September 30, 2013**

Name: Solutions.

Directions: Show all work. No credit for answers without work.

1. [**2.5 points**] Find the point at which the line $x = 3 - t$, $y = 2 + t$, $z = 5 + t$ intersects the plane $x - y + 2z = 9$, or show there is no point of intersection.

$$x - y + 2z = 9$$

$$\langle -1, 1, 1 \rangle$$

$$\langle 1, -1, 2 \rangle$$

$$(3-t) - (2+t) + 2(5+t) = 9$$

$$(3\cancel{-t}) - (2\cancel{+t}) + 10 + \cancel{2t} = 9$$

$$11 = 9$$

So there is no intersection.

2. [**2.5 points**] Find an equation for the plane containing points $(1, 1, 1)$, $(2, -1, 3)$, and $(5, -3, -2)$.

$$\vec{a} = \langle 2, -1, 3 \rangle - \langle 1, 1, 1 \rangle = \langle 1, -2, 2 \rangle$$

$$\vec{b} = \langle 5, -3, -2 \rangle - \langle 1, 1, 1 \rangle = \langle 4, -4, -3 \rangle$$

Find Normal vector:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 2 \\ 4 & -4 & -3 \end{vmatrix} = (6+8)\vec{i} - (-3-8)\vec{j} + (-4+8)\vec{k}$$

$$= \langle 14, 11, 4 \rangle$$

So: $\boxed{14x + 11y + 4z = 29}$

3. **[2.5 points]** Find the unit tangent vector $\vec{T}(t)$ to the curve $\vec{r}(t) = (\cos t)\vec{i} + (3t)\vec{j} + (2\sin(2t))\vec{k}$ at the point $t = 0$.

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \frac{\langle -\sin(t),\ 3,\ 4\cos(2t)\rangle}{\sqrt{\sin^2(t) + 9 + 16\cos^2(2t)}}$$

$$\vec{T}(0) = \frac{\langle 0, 3, 4\rangle}{\sqrt{0+9+16}} = \boxed{\left\langle 0, \tfrac{3}{5}, \tfrac{4}{5}\right\rangle}$$

4. **[2.5 points]** Find the curvature $\kappa(t)$ of $\vec{r}(t) = t^2\vec{i} + t\vec{k}$.

$$\vec{r}\,'(t) = 2t\vec{i} + \vec{k}$$

$$\vec{r}\,''(t) = 2\vec{i}$$

$$\vec{r}\,' \times \vec{r}\,'' = \begin{vmatrix} i & j & k \\ 2t & 0 & 1 \\ 2 & 0 & 0 \end{vmatrix} = 0\vec{i} - (0-2)\vec{j} + 0\vec{k} = 2\vec{j}.$$

$$K(t) = \frac{|r' \times r''|}{|r'|^3} = \frac{2}{\left(\sqrt{4t^2+1}\right)^3} = \frac{2}{(4t^2+1)^{3/2}}$$