Math251 **Quiz 2** **September 4, 2013**

Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[1 point]** Complete the following sentence: The number of solutions to a system of linear equations is either ....

   0, 1, or infinite.

2. **[1 point]** Give an example of an *inconsistent* system of linear equations in a single variable x.

   $$x = 3$$
   $$5x = 4$$

   Many other answers are possible

3. **[4 points]** Find all solutions to the following system of linear equations. (Use any method you like.)

$$\begin{array}{rcrcrcr} 4x & + & y & & & = & -2 \\ -2x & + & y & + & z & = & 7 \\ 2x & + & y & + & z & = & 1 \end{array}$$

$$\left[\begin{array}{ccc|c} 4 & 1 & 0 & -2 \\ -2 & 1 & 1 & 7 \\ 2 & 1 & 1 & 1 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ -2 & 1 & 1 & 7 \\ 4 & 1 & 0 & -2 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 0 & 2 & 2 & 8 \\ 0 & -1 & -2 & -4 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 0 & 1 & 2 & 4 \\ 0 & 2 & 2 & 8 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & -1 & -3 \\ 0 & 1 & 2 & 4 \\ 0 & 2 & 2 & 8 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & -1 & -3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -2 & 0 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & -1 & -3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 0 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 0 & 0 & -3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3/2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\boxed{\text{So } x = -3/2,\ y = 4,\ z = 0 \text{ is the only solution.}}$$

4. **[4 points]** Using *matrices* and *Gauss-Jordan* elimination, find all solutions to the following system of linear equations.

$$\begin{array}{rcrcrcrcr} 3x_1 & - & 9x_2 & + & x_3 & & & = & 10 \\ x_1 & - & 3x_2 & & & & & = & 1 \\ 2x_1 & - & 6x_2 & - & x_3 & + & x_4 & = & -1 \\ & & & - & 2x_3 & + & x_4 & = & -10 \end{array}$$

$$\left[\begin{array}{cccc|c} 3 & -9 & 1 & 0 & 10 \\ 1 & -3 & 0 & 0 & 1 \\ 2 & -6 & -1 & 1 & -1 \\ 0 & 0 & -2 & 1 & -10 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -3 & 0 & 0 & 1 \\ 3 & -9 & 1 & 0 & 10 \\ 2 & -6 & -1 & 1 & -1 \\ 0 & 0 & -2 & 1 & -10 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & -3 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & -1 & 1 & -3 \\ 0 & 0 & -2 & 1 & -10 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -3 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & -3 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 \quad x_2 \quad x_3 \quad x_4$

free

$$\boxed{\begin{array}{l} \text{So the solutions are} \\ x_1 = 1+3c \\ x_2 = c \\ x_3 = 7 \\ x_4 = 4 \\ \text{where } c \in \mathbb{R}. \end{array}}$$