Name: Solutions

**Directions:** Show all work. No credit for answers without work.

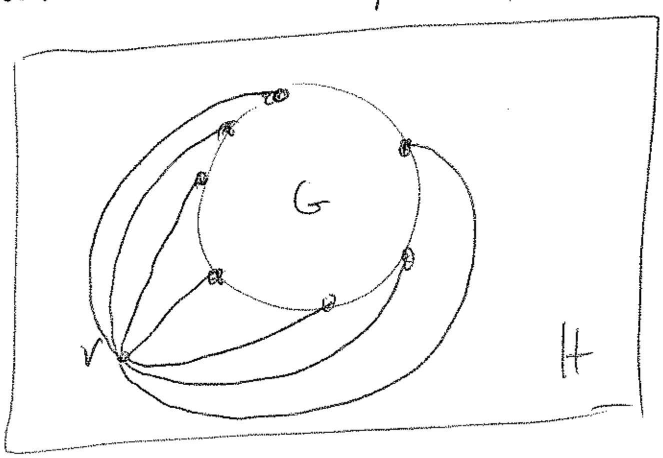
1. [**4 points**] Recall:

   - A graph is *outerplanar* if it can be drawn in the plane without edge crossings so that all vertices are on the outer face.
   - The vertices of every planar graph can be colored with 4 colors so that adjacent vertices receive different colors. (So, every map requires at most 4 colors.)

   Prove that the vertices of every outerplanar graph can be colored with 3 colors so that adjacent vertices receive different colors. *Hint:* given an outerplanar graph $G$ that we want to color, use $G$ to make a planar graph $H$ in such a way that a 4-coloring of the vertices of $H$ will give a 3-coloring of the vertices of $G$.

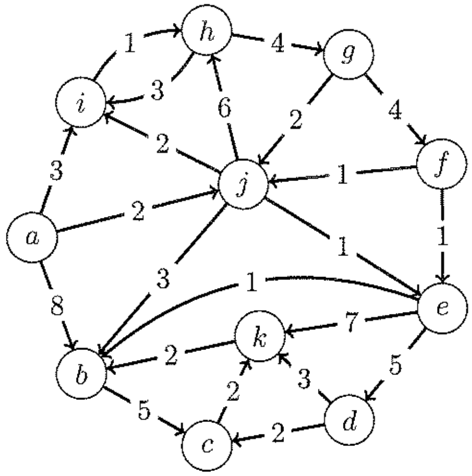
Let H be the graph obtained from G by adding a new vertex v that is adjacent to all vertices in G.

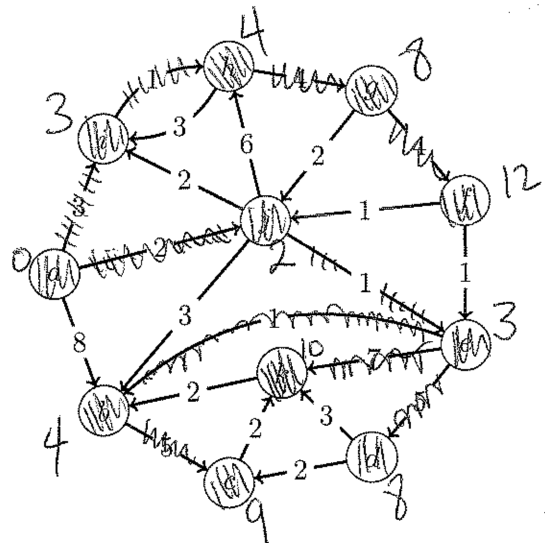
Because G is outerplanar, H is planar:




We know that H has a 4-coloring. Because v is adjacent to every vertex in G, the color used on v is not used on any vertex in G. Therefore the same coloring gives a 3-coloring of G in which adjacent vertices receive distinct colors. ☒

2. [**2 parts, 3 points each**] Consider the following directed graph. (Two copies for your convenience.)




(a) Use Dijkstra's algorithm to find the distance from $a$ to all other vertices.

| a | b | c | d | e | f | g | h | i | j | k |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 4 | 9 | 8 | 3 | 12 | 8 | 4 | 3 | 2 | 10 |

(b) Give a shortest path from $a$ to $c$.

a j e b c , distance 9.