Name: _____Solutions_____

Directions: Show all work. No credit for answers without work. Unless specifically asked for a numerical answer, you may leave your answers in terms of exponentials, factorials, permutation numbers, and binomial coefficients.

1. **[2 parts, 5 points each]** Prove or find a counterexample for the following. Recall that a graph G is *k-regular* if every vertex in G has degree k.

(a) If G is a 4-regular graph, then the number of vertices in G is odd.

False: Counterexample:
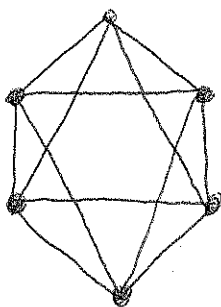
6 vertices

4-regular

(b) If G is a 5-regular graph, then the number of vertices in G is even.

True. Proof: Let G be a 5 regular graph with n vertices and m edges. By the degree sum formula,

$$\sum_{v \in V(G)} \deg(v) = 2m$$

$$5n = 2m$$

Therefore $5n$ is divisible by 2, and it follows that n is even.