

Name: Key

Directions: Show all work. No credit for answers without work. Unless specifically asked for a numerical answer, you may leave your answers in terms of exponentials, factorials, permutation numbers, and binomial coefficients.

1. [4 points] Determine the number of non-negative integral solutions to the following.

(a) $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 80$, with $x_3 \geq 8$.

~~21~~ # solns to $x_1 + \dots + x_6 = 72$; $x_1, x_2, \dots, x_6 \geq 0$

$\Rightarrow 72$ stars, 5 bars $\Rightarrow \boxed{\binom{77}{5}} = \text{~~19,692,035~~ 11,975,815}$

(b) $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 80$, with $x_3 \geq 8$ and $x_5 \leq 50$.

Take (a) and subtract #solns with $x_3 \geq 8$ and $x_5 \geq 51$:

solns to $x_1 + \dots + x_6 = 21$, $x_1, \dots, x_6 \geq 0$

$\Rightarrow 21$ stars, 5 bars $\Rightarrow \binom{26}{5}$

Answer: $\boxed{\binom{77}{5} - \binom{26}{5}}$

19,692,035
~~19,692,035~~
~~11,975,815~~

2. [3 points] A company wishes to order s sandwiches for their annual party from a menu that lists k types of sandwich. How many ways are there for the company to complete its order?

• s stars
• $k-1$ bars

$\boxed{\binom{s+k-1}{k-1}}$

or

$\boxed{\binom{s+k-1}{s}}$

3. [3 points] Find $\mathcal{P}(\{1, 2, 3\})$.

$\mathcal{P}(\{1, 2, 3\}) = \boxed{\{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}}$

Note: this question asks for the set $\mathcal{P}(\{1, 2, 3\})$,
not its size $|\mathcal{P}(\{1, 2, 3\})|$.