Name:

Directions: Show all work. No credit for answers without work. Unless specifically asked for a numerical answer, you may leave your answers in terms of exponentials, factorials, permutation numbers, and binomial coefficients.

- 1. [4 points] Determine the number of non-negative integral solutions to the following.
  - (a)  $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 80$ , with  $x_3 \ge 8$ .

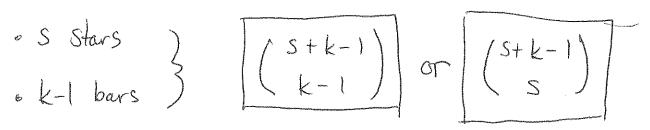
# solus to 
$$x_1 + \dots + x_6 = 72$$
;  $x_{1,1}x_{2,1} \dots x_6 \ge 0$ 
 $\Rightarrow 72 \text{ stors}, 5 \text{ bars} \Rightarrow \boxed{(75)} = \boxed{(9,757,815)}$ 

(b)  $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 80$ , with  $x_3 \ge 8$  and  $x_5 \le 50$ .

Take (a) and subtract #solus with  $x_3 \ge 8$  and  $x_5 \ge 51$ :

# solus to  $x_1 + \cdots + x_k = 21$ ,  $x_1, \cdots, x_k \ge 0$   $\Rightarrow 21$  stars, 5 bars  $\Rightarrow (\frac{26}{5})$ Answer:  $(\frac{72}{5}) - (\frac{26}{5})$   $\Rightarrow 10, 100$ 

2. [3 points] A company wishes to order s sandwiches for their annual party from a menu that lists k types of sandwich. How many ways are there for the company to complete its order?



3. [3 points] Find  $\mathcal{P}(\{1,2,3\})$ .

Note: this question asks for the set  $P({\{1,2,3\}})$ , not its size  $|P({\{1,2,3\}})|$ .