Name: _Key_

Directions: Show all work. No credit for answers without work. Unless specifically asked for a numerical answer, you may leave your answers in terms of factorials, permutation numbers, and binomial coefficients.

1. Evaluate $\binom{8}{4}$ numerically.

$$\binom{8}{4} = \frac{8!}{4! \cdot 4!} = \frac{8 \cdot 7 \cdot \overset{2}{6} \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = \boxed{70}$$

2. The DMV offers a 10 question multiple choice test. Each question has 4 choices, consisting of 1 correct choice and 3 incorrect choices. All questions must be answered.

   (a) How many ways are there to complete the test?

   $$4 \cdot 4 \cdot 4 \cdot \cdots \cdot 4 = \boxed{4^{10}} = \boxed{1,048,576}$$

   (b) To pass, at least 7 questions must be answered correctly. How many ways are there to pass the test?

   $$\boxed{\binom{10}{7} 3^3 + \binom{10}{8} 3^2 + \binom{10}{9} 3 + \binom{10}{10} \cdot 1}$$

   $$= 3240 + 405 + 30 + 1 = \boxed{3,676}$$

   or about 0.35% of all ways to complete the test.

3. Let n be a positive integer.

   (a) Express $\binom{n}{0} + 3\binom{n}{1} + 3^2\binom{n}{2} + \cdots + 3^n\binom{n}{n}$ using sigma notation.

   $$\sum_{k=0}^{n} 3^k \binom{n}{k}$$

   (b) Evaluate the sum. Justify your answer.

   Since $\displaystyle\sum_{k=0}^{n} 3^k \binom{n}{k} = \sum_{k=0}^{n} \binom{n}{k} 3^k 1^{n-k}$, by binom. thm, this is

   $$(3+1)^n = \boxed{4^n}.$$