Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[2 parts, 2 points each]** Consider the group $(\mathbb{Z}_7^*, \odot)$ where $x \odot y = xy \mod 7$.

   (a) Construct the operation table for the group.

$\odot$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

   (b) What is the inverse of 3?

Since $3 \odot 5 = 1$, $\boxed{5}$ is the inverse of 3.

2. Let $W = Z_2^3$ and let $E\colon W \to \mathbb{Z}_2^9$ be the encoding function which repeats a message 3 times, so that $E(w) = www$. Let $D\colon \mathbb{Z}_2^9 \to W$ be the corresponding decoding function. The channel flips bits with probability $p = 0.20$.

   (a) **[1 point]** If the received codeword is $r = 100110010$, find the decoded message $D(r)$.

   $D(r) = 110$

   (b) **[1 point]** In part (a), what can you say about the number of errors that occurred during transmission?

   At least two bits are flipped in transmission.

   (c) **[2 points]** If a message w in W is transmitted *without* encoding, what is the probability that w is received without error?

   $$(1-p)^3 = (0.8)^3 \approx \boxed{0.512}$$

   (d) **[2 points]** What is the probability that a transmission is successful with the encoding scheme?

   Prob(first bit decoded correctly) = Pr(≤ 1 bit flip in $\overset{\downarrow}{a}\_\_\overset{\downarrow}{b}\_\_\overset{\downarrow}{c}\_\_$)

   $$= (1-p)^3 + 3p(1-p)^2 \approx 0.896$$

   Pr(all bits decoded correctly) $= \left((1-p)^3 + 3p(1-p)^2\right)^3$

   $$= (0.896)^3 \approx \boxed{0.7193}$$