Name: Solutions

Directions: Show all work. No credit for answers without work.

1. Let V be the set of all positive real numbers; define $\oplus$ so that $\mathbf{u} \oplus \mathbf{v} = \mathbf{uv}$ (standard multiplication of reals) and $\odot$ so that $c \odot \mathbf{u} = \mathbf{u}^c$. It can be shown that V is a vector space.

   (a) [**1 point**] Identify the zero vector of V. (Your answer should be a specific element in V.)

   We know: $\vec{0} = 0 \odot u = u^0 = \boxed{1}$. Also, 1 is the only vector x for which $u \oplus x = u$ and $x \oplus u = u$.

   (b) [**3 points**] Prove that $c \odot (\mathbf{u} \oplus \mathbf{v}) = (c \odot \mathbf{u}) \oplus (c \odot \mathbf{v})$.

Note that $c \odot (u \oplus v) = c \odot uv = (uv)^c$, and

$(c \odot u) \oplus (c \odot v) = (u^c) \oplus (v^c) = u^c v^c$.

Since $(uv)^c = u^c v^c$, it follows that

$$c \odot (u \oplus v) = (c \odot u) \oplus (c \odot v). \qquad \blacksquare$$

2. [**2 parts, 3 points each**] Determine if the given subsets of $\mathbb{R}^3$ are subspaces. Justify your answers.

   (a) $W = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a + b + c = 1 \right\}$

   Not a subspace:

   $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \in W$ but $5 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} \notin W$

   (b) $W = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : abc = 0 \right\}$

   Not a subspace:

   $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \in W$, $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \in W$ but

   $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \notin W$.