

Name: Solutions

Directions: Show all work. No credit for answers without work.

1. [3 points] Find the number of inversions in the permutation 5614273.



11

 inversions total

2. Evaluate the following determinants.

(a) [4 points]  $\det \begin{pmatrix} 5 & -2 & 1 \\ 0 & 1 & -1 \\ 2 & 3 & 1 \end{pmatrix}$ .

$$5 + (-2)(-1)2 + 0 - (2 + 3(-1)(5) + 0)$$

$$= 5 + 4 - (2 - 15) = 9 + 13 = \boxed{22}$$

(b) [3 points]  $\det \begin{pmatrix} 0 & 0 & \textcircled{2} & 0 & 0 \\ 7 & 0 & 1 & 0 & \textcircled{-1} \\ 2 & \textcircled{1} & 4 & 0 & 3 \\ -2 & 2 & 1 & \textcircled{2} & 4 \\ \textcircled{3} & 0 & -3 & 0 & 0 \end{pmatrix}$ .

Only one permutation contributes:  $\pi = 35241$   
 # inversions = 7,  $\text{sgn}(\pi) = -1$ .

$$\text{So } \det([ \cdot ]) = (-1) \cdot 3 \cdot 1 \cdot 2 \cdot 2 \cdot (-1)$$

$$= \boxed{12}$$