Math343 **Quiz 10** **November 2, 2012**

Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[5 points]** Determine whether $\left\{ \begin{bmatrix} 2 \\ 2 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ -5 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \\ -9 \end{bmatrix} \right\}$ is linearly independent in $\mathbb{R}^4$.

$$\begin{bmatrix} 2 & 2 & 6 \\ 2 & 5 & -3 \\ -1 & -5 & 9 \\ -1 & 1 & -9 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 1 & 3 \\ 0 & 3 & -9 \\ 0 & -4 & 12 \\ 0 & 2 & -6 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

free variable

Therefore these vectors are $\boxed{\text{linearly dependent.}}$

2. **[5 points]** Determine whether $\left\{ \begin{bmatrix} 1 \\ -5 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -5 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix} \right\}$ is a base for $\mathbb{R}^4$.

$$\begin{vmatrix} 1 & 2 & 1 & 1 \\ -5 & 1 & 1 & 1 \\ 3 & -5 & -1 & 1 \\ -2 & 1 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 1 & 1 \\ 0 & 11 & 6 & 6 \\ 0 & -11 & -4 & -2 \\ 0 & 5 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 1 & 1 \\ 0 & 11 & 6 & 6 \\ 0 & 0 & 2 & 4 \\ 0 & 5 & 3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 11 & 6 & 6 \\ 0 & 2 & 4 \\ 5 & 3 & 1 \end{vmatrix} = (11\cdot 2\cdot 1 + 6\cdot 4\cdot 5 + 6\cdot 0\cdot 3) - (5\cdot 2\cdot 6 + 3\cdot 4\cdot 11 + 1\cdot 0\cdot 6)$$

$$= (22 + 120 + 0) - (60 + 132)$$

$$= -50$$

Since determinant is $\neq 0$, this $\boxed{\text{is a basis}}$ for $\mathbb{R}^4$.