

Show all work.

① [14 points] Find the inverse of $\begin{bmatrix} 1 & 0 & 2 \\ -3 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$

② [14 points] Find a matrix in reduced row echelon form which is row equivalent to the matrix at the right.

$$\begin{bmatrix} 1 & 2 & -2 & 4 \\ 2 & 4 & -1 & 17 \\ -1 & -2 & 3 & -1 \end{bmatrix}$$

③ [10 points] In solving a 4×4 linear system (unknowns x_1, x_2, x_3, x_4) a student gets the matrix at the right. Find the solution for the system.

$$\begin{bmatrix} 1 & 8 & 0 & -3 & 6 \\ 0 & 0 & 1 & 4 & 7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

④ [14 points] Compute the determinants. Explain answers.

(a) $\begin{bmatrix} 5 & 3 & 8 \\ 0 & -2 & 6 \\ 0 & 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 19 & 2 & 1 \\ 7 & 1 & 7 & 2 \\ 3 & 13 & 3 & 4 \\ 4 & 9 & 4 & 81 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 & 0 & -2 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 5 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \end{bmatrix}$

⑤ [10 points] What are the possibilities for the number of solutions for a linear system with 25 equations and 28 unknowns? Explain briefly.

⑥ [20 points] Let $B = \begin{bmatrix} 8 & 0 \\ 3 & 1 \end{bmatrix}$

(a) Find elementary matrices D and E such that $DEB = I_2$

(b) Express B and B^{-1} as products of elementary matrices.

(c) Find all real numbers c such that $\begin{bmatrix} 8 & 4 \\ 5 & c \end{bmatrix}$ can be expressed as a product of elementary matrices. Give a one sentence explanation.

[This part should not take much time or writing.]

⑦ [10 points] Suppose C is a 12 by 16 matrix whose 5th row and 7th column are all 0's. If A is a 19 by 12 matrix, what can you say about the number of 0's in AC ? Explain. [Be sure to answer the question.]

⑧ [8 points] Does there exist a 4×4 matrix A such that $Ax = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$ has a unique solution and $Ax = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ has no solution? Explain briefly. [Should not take much time or writing.]

Extra Credit Optional take-home problem. Ground-rules:

No giving or getting help. If you do it, write the pledge "I have neither given nor received help" and sign it.

⑨ A curve in the xy -plane of the form:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

where A, B, C, D, E, F are constants, is called a conic curve. For example, if $A=1, B=0, C=0, D=5, E=1, F=9$ you get a parabola; if $A=C=D=E=0, B=1, F=7$ you get an hyperbola. Show that given any 5 points in the xy -plane there exists a conic curve through those 5 points.