

1. Implicit Differentiation

(a) Find $\frac{dy}{dx} = y'(x)$ for $\sin^{-1}(xy) = x^2 - 3$.

(b) Find $\frac{dy}{dx} = y'(x)$ for $y^x + 3 \tan(y) = 12$.

2. **What fails?** For each of the following, determine whether we can apply the Mean Value Theorem. If we cannot apply the Mean Value Theorem, describe why. If we can apply the Mean Value Theorem, state the conclusion.

(a) $f(x) = x^2 + \sin(x)$ in $[0, \pi/2]$

(b) $f(x) = 3x^2 + \tan(x)$ in $[0, \pi/2]$

(c) $f(x) = \frac{x^3+27}{x+3}$ in $[-5, 5]$.

(d) $f(x) = x^{1/3}$ in $[-1, 1]$

3. [2.9, 9-16] Determine whether the function is increasing, decreasing or neither.

(a) $f(x) = x^5 + 3x^3 - 1$

(b) $f(x) = x^4 + 2x^2 + 1$

4. [2.9, 20] Prove that $x^4 + 6x^2 - 1 = 0$ has exactly two solutions.

*5. [2.9, 26] Two runners start a race at time 0. At some time $t = a$, one runner has pulled ahead, but the other runner has taken the lead by time $t = b$. Prove that at some time $t = c > 0$, the runners were going exactly the same speed. Does there have to be a time between $t = a$ and $t = b$ during which the runners are traveling the same speed?

*6. [2.9, 40] Show that for any real numbers u and v , $|\cos(u) - \cos(v)| \leq |u - v|$.