

The exam covers Ch3.8, 4.1-4.6, and 5.1-5.3. **My personal advice: be sure you can define the definite integral and state both parts of the fundamental theorem of calculus; these are very likely to be on the exam.** See also Prof. Mortensen's course review sheet on the course website.

### Ch3.8: Related Rates

- (# 8) Suppose a forest fire spreads in a circle with radius changing at a rate of 5 feet per minute. When the radius reaches 200 feet, at what rate is the area of the burning region increasing?
- (# 9) A 10-foot ladder leans against the side of a building. If the bottom of the ladder is pulled away from the wall at the rate of 3 ft/s and the ladder remains in contact with the wall, find the rate at which the top of the ladder is dropping when the bottom is 6 feet from the wall.

### Ch4.1: Antiderivatives

- Find the general antiderivative:

(a)  $\int \frac{x + 2x^{3/4}}{x^{5/4}} dx$

(b)  $\int \frac{4}{\sqrt{1-x^2}} dx$

(c)  $\int \frac{e^x}{e^x + 3} dx$

- Find the function satisfying  $f''(x) = 2x$ ,  $f'(0) = -3$ ,  $f(0) = 2$ .
- Find all functions satisfying  $f'''(x) = \sin(x) - e^x$ .

### Ch4.2: Sums and Sigma Notation

On the exam, you will be given these formulas from p. 356.

If  $n$  is any positive integer and  $c$  is any constant, then

1.  $\sum_{i=1}^n c = cn$  (sum of constants),

2.  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$  (sum of the first  $n$  positive integers), and

3.  $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$  (sum of the squares of the first  $n$  positive integers).

1. Compute  $\sum_{i=1}^{250} (i^2 + 8)$ .

2. Compute the sum and the limit of the sum as  $n \rightarrow \infty$ :  $\sum_{i=1}^n \frac{1}{n} \left[ 4 \left( \frac{2i}{n} \right)^2 - \left( \frac{2i}{n} \right) \right]$ .

### Ch4.3: Area

- Use Riemann sums and a limit to compute the exact area under the curve:  $y = x^2 + 3x$  on  $[0, 1]$ . **You may not use the fundamental theorem of calculus.** In other words, evaluate the definite integral

$$\int_0^1 x^2 + 3x dx \text{ from the definition.}$$

2. True or false: if  $f(x)$  is increasing on  $[a, b]$ , then a Riemann sum with right-endpoint evaluation will give you a value that is at least as large as the signed area under the curve  $f(x)$  on  $[a, b]$ .

#### Ch4.4: The Definite Integral

1. State the integral mean value theorem.
2. Give upper and lower bounds on  $\int_{-1}^1 \frac{3}{x^3 + 2} dx$ .

#### Ch4.5: The Fundamental Theorem of Calculus

1. State:
  - (a) Part I of the fundamental theorem of calculus
  - (b) Part II of the fundamental theorem of calculus
2. Compute the following.

(a)  $\int_0^2 (\sqrt{x} + 1)^2 dx$

(b)  $\int_0^{\pi/3} \frac{3}{\cos^2(x)} dx$

(c)  $\int_1^2 \frac{x^2 - 3x + 4}{x^2} dx$

3. Find  $f'(x)$ :  $f(x) = \int_{x^3}^{x^2} \sin(\sqrt{t}) dt$ .

#### Ch4.6: Integration by Substitution

1. Evaluate the indefinite integrals:

(a)  $\int \sin^3 x \cos x dx$

(b)  $\int \frac{x}{x^2 + 4} dx$

(c)  $\int \frac{2x + 3}{x + 7} dx$

(d)  $\int \frac{4}{x(\ln x + 1)^2} dx$

2. Evaluate the definite integrals:

(a)  $\int_1^e \frac{\ln x}{x} dx$

(b)  $\int_0^2 x\sqrt{x^2 + 1} dx$

(c)  $\int_0^2 x\sqrt{x + 1} dx$

**Ch5.1: Area Between Curves**

Sketch and find the area of the region bounded by the given curves. Choose the variable of integration so that the area is written as a single integral.

1.  $x = y$ ,  $x = -y$ ,  $x = 1$
2.  $y = 3x$ ,  $y = 2 + x^2$

**Ch5.2-5.3: Volumes**

Compute the volume:

1. The region bounded by  $y = x^2$  and  $y = 2 - x^2$ , revolved about  $x = 2$ .
2. The region bounded by  $x = y^2$  and  $x = 2 + y$ , revolved about  $x = -1$ .

**General Review**

1. (a) Define the indefinite integral.  
(b) Define the definite integral.  
(c) How are these the two concepts related?