

1. True/False.

- (a) Every continuous function has a derivative.
- (b) If $f(x)$ and $g(x)$ have a derivative at $x = a$, then $h(x) = f(x)g(x)$ also has a derivative at $x = a$.
- (c) If $f(x)$ and $g(x)$ have a derivative at $x = a$, then $h(x) = \frac{f(x)}{g(x)}$ also has a derivative at $x = a$.
- (d) If $f(x)$ and $g(x)$ have a derivative at $x = a$, then $h(x) = f(g(x))$ also has a derivative at $x = a$.
- (e) If $f(x)$ is continuous at $x = a$, then $f(x)$ is differentiable at $x = a$.
- (f) If $f(x)$ is differentiable at $x = a$, then $f(x)$ is continuous at $x = a$.
- (g) If $f(0) = 12$, then $\lim_{x \rightarrow 0} f(x) = 12$.
- (h) If $f(0) = 12$ and $f(x)$ is continuous, then $\lim_{x \rightarrow 0} f(x) = 12$.
- (i) $\lim_{x \rightarrow 0} \frac{1}{x^2}$ does not exist.
- (j) $\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$.

2. Give an example of each of the following.

- (a) A function which has a removable discontinuity at $x = 0$.
- (b) A function which is continuous but not differentiable at $x = 0$.
- (c) A function for which $f'(x) = 2$ for all x and $f(1) = 1$.
- (d) Two different functions with the property that $f'(x) = f(x)$ for all x .
- (e) A function with the property that $f''(x) = f'(x)$ but $f'(x)$ is not the same as $f(x)$.
- (f) A function for which $f(x) \geq 0$ for all x and $f'(x) < 0$ for all x .
- (g) A function for which $f'(x)$ is not always zero but $f'(x) = 0$ for infinitely many x .

3. State the definition of each of the following.

- (a) $f(x)$ is continuous at $x = a$.
- (b) $f(x)$ has a removable discontinuity at $x = a$.
- (c) The derivative of $f(x)$ at $x = a$.

4. State the following theorems.

- (a) Squeeze Theorem
- (b) Intermediate Value Theorem

5. Evaluate. Answer with a number, $-\infty$, ∞ , or “does not exist”. Do not use L'Hopital's rule.

(a) $f(x) = \begin{cases} x^2 & x \neq 0 \\ 4 & x = 0 \end{cases}$, $g(x) = 2x$. $\lim_{x \rightarrow 0} f(g(x))$

(b) $f(x) = \begin{cases} x^2 & x \neq 0 \\ 4 & x = 0 \end{cases}$, $g(x) = 2x$. $f\left(\lim_{x \rightarrow 0} g(x)\right)$

(c) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 + x - 2}$

(d) $\lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{1 - x}$

(e) $\lim_{x \rightarrow 0} x^{1/3} \sin(4/x)$

(f) $\lim_{x \rightarrow 0} x^{1/2} \sin(4/x)$

(g) $\lim_{x \rightarrow \pi^-} \csc(x)$

(h) $\lim_{x \rightarrow -\infty} \frac{-x^2 - 4x + 8}{3x^3}$

(i) $\lim_{x \rightarrow -\infty} \frac{-x^2 - 4x + 8}{3x^2}$

(j) $\lim_{x \rightarrow 0^-} \frac{x^2 - 4x + 8}{3x^3}$

(k) $\lim_{x \rightarrow 0^+} \frac{x^2 - 4x + 8}{3x^2}$

(l) $\lim_{x \rightarrow 0} e^{1/x}$

(m) $\lim_{x \rightarrow 0^-} e^{1/x}$

(n) $\lim_{t \rightarrow 0} \frac{t}{\sin(2t)}$

(o) $\lim_{x \rightarrow 0} \frac{\cos(4x^2) - 1}{x^2}$

6. Find the derivative for each of the following functions.

(a) $f(x) = \frac{e^x}{\cos(x)}$

(b) $f(x) = x^2 \cot(x)$

(c) $f(x) = \frac{\ln(x)}{\cos(x^2)}$

(d) $f(x) = \sec(e^{5 \cos(x^2)})$

(e) $f(x) = \tan \sqrt{x^2 + 1}$

(f) $f(x) = 4x^2 \sin(x) \sec(3x)$

(g) $f(x) = (\csc(x))^4$

(h) $f(x) = 3^{\tan(x)}$

(i) $f(x) = (\cos(x))^{\sin(x)}$

7. Find $f''(x)$ for each of the following functions.

(a) $f(x) = 2x^3 - 4x^2 + 5x + 2$

(b) $f(x) = e^{\sin(x)}$