

CSTBC Homework 11

June 29, 2007

1 Equality

In Lecture 11, we saw that if T is a binary tree and $L = \{v \in V(T) \mid v \text{ has no children}\}$ is the set of all leaves in T , then

$$\sum_{v \in L} 2^{-\text{depth}(T)} \leq 1.$$

Describe the binary trees T for which it is the case that

$$\sum_{v \in L} 2^{-\text{depth}(T)} = 1.$$

Your description should be as simple as possible; for example, it should not involve any summations or recursive definitions. Prove that your description is correct.

2 Eulerian Trails

In Lecture 11, we saw that a graph G contains an Eulerian circuit if and only if G has at most one component with edges and all vertices in G have even degree.

Develop an analogous theorem to characterize the graphs which contain Eulerian trails. That is, complete the statement “A graph G contains an Eulerian trail if and only if ...” and prove your statement.